Appendix A

Structure of reconstruction matrices

The motion-corrupted magnetization during the c^{th} readout is denoted $m_c(\mathrm{r}) = |m_c(r)|e^{i\phi_c(r)}$.

The phase corruption during the c^{th} excitation is expressed by the $N^2 \times N^2$ matrix $\mathbf{P^c}$:

$$\mathbf{P^c} = \begin{pmatrix} e^{i\phi_c(r_1)} & 0 & \dots & 0 \\ 0 & e^{i\phi_c(r_2)} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & e^{i\phi_c(r_{N^2})} \end{pmatrix} \quad (7)$$

$\mathbf{P^c m}$ is the phase-corrupted magnetization that is the source if signal during the c^{th} readout. Each of the C interleaves has a different phase corruption, represented by stacking the $\mathbf{P^c}$'s to form $\mathbf{P}$:

$$\mathbf{P} = \begin{pmatrix} P^1 \\ P^2 \\ \vdots \\ P^C \end{pmatrix} \quad (8)$$

$\mathbf{P}$ splits the magnetization vector m into C separate phase-corrupted magnetization vectors. These vectors are stacked to form the $CN^2 \times 1$ vector $\mathbf{P}$m representing the phase-corrupted magnetization that is sampled over the course of the experiment.

Each of the phase-corrupted magnetization vectors is separately Fourier transformed by the matrix $\mathbf{F}$ ($CN^2 \times CN^2$):

$$\mathbf{F} = \begin{pmatrix} F^N & 0 & \dots & 0 \\ 0 & F^N & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & F^N \end{pmatrix} \quad (9)$$

$\mathbf{F}$ is composed of C 2DFT matrices $\mathbf{F^N}$ ($N^2 \times N^2$) consisting of the Fourier kernels:

$$\mathbf{F^N}_{lm} = e^{i2\pi k_l \cdot r_m} \quad (10)$$

The product FPm is the $CN^2 \times 1$ vector representing the C phase-corrupted spectra, sampled in Cartesian coordinates.

Finally, the spectra given by $\mathbf{FPm}$ are resampled onto the k-space sampling trajectories by the resampling matrix $\mathbf{G}$ ($CR \times CN^2$):

$$\mathbf{G} = \begin{pmatrix} G^1 & 0 & \dots & 0 \\ 0 & G^2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & G^C \end{pmatrix} \tag{11}$$

Each submatrix $\mathbf{G}^c$ ($R \times N^2$) interpolates the Cartesian k-space points $\mathbf{k}_l$ onto the sampling points for the c^{th} readout s_m^c :

$$\mathbf{G}^c{}_{lm} = \text{sinc}(\mathbf{k}_l \,.\, s_m^c) \tag{12}$$

Note that for trajectories that collect data on Cartesian coordinates (such as spinwarp or interleaved EPI), the elements of $\mathbf{G}$ are discrete delta functions.

APPENDIX B

The mixing matrix

Using the notation of Appendix A, the elements of the mixing matrix M for an arbitrary trajectory are:

$$\mathrm{M}_{lm} = \sum_{c=1}^{C} e^{i(\phi_c(r_m)-\phi_c(r_l))} \sum_{r=1}^{R} \left[\sum_{n=1}^{N^2} \sin c\left(k_n - s_r^c\right) e^{+i2\pi k_n \cdot r_m} \right] \left[\sum_{n=1}^{N^2} \sin c\left(k_n - s_r^c\right) e^{-i2\pi k_n \cdot r_l} \right] \quad (13)$$

The summation over each readout *r* described non-idealities in the *k*-space trajectory that causes coupling between voxels within a readout. The phase difference describes the aliasing caused by motion-induced phase errors. These phase offsets are manifested as perturbations of the encoding functions used in the trajectory.